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"Instability at the face: its repercussions for tunnelling technology".

Instability at the face: its repercussions for tunnelling technology

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During the tunnel construction process, especially in shallow tunnels, instability of the excavation face frequently occurs although in this area stability is assisted by confinement of the ground which gives it a state of stress in triaxial compression. In these cases the dome effect either does not occur, or its duration is so fleeting that it does not guarantee stability.

Construction techniques

Whenever this phenomenon becomes evident in rock tunnels in portal areas or in fault zones, different preliminary support techniques are used:

1. Excavation in stages with a stabilising central core;
2. Ground freezing;
3. Grouting;
4. Prior installation of spiles in the excavation perimeter, slightly slanting in relation to the tunnel axis, forming an open beam (umbrella);
5. a) using ordinary micropiles or coupling-pipes
   b) using jet grouting.

Technique (1) is frequently applied combined with one of the other techniques.

The different types of shield machine were developed to try to guarantee stability both at the excavation face and at the back of the shield by placing the lining immediately. Their use is appropriate when tunnels are to be built through ground with bad geomechanical characteristics.

Rock which shows the greatest tendency towards instability is: shale, and clayey schist, rock with abundant minerals like kaolin, illite, sericite, chlorite and graphite, micaceous rock which is highly fractured and altered, unconsolidated soft rock and very altered pyroclastic deposits. Soils tending to instability include: soils not very compact — clays, silts, sands or un-cemented sandstones in the presence of water, and mixtures of these soils.

Causes of instability

1. Attributable to rock:
   a) Physical-mechanical characteristics of the minerals making up the rock matrix;
   b) Mechanical defects of the rock;
2. State of stress;
3. Presence of water

1a. Rock characteristics

Rock most prone to instability is that which has undergone greatest meteorisation or alteration as at the portal area, in shallow tunnels with little overburden. Chemical alteration occurs in those minerals with an abundance of magnesium, calcium or iron, which are most prone to meteorisation. Similar minerals which are the product of alteration give very fine grained clay and micaceous (sericite), apart from aluminium and iron hydroxides, which are the most resistant.

Meteorisation caused by the movement of underground water produces clayey minerals like montmorillonite, illite, chlorite, vermiculite and clayey mixtures. Through the hydrothermal process, argilisation is produced which converts rock, with the exception of quartz, into clayey mineral aggregates.

All these processes can convert competent rock into rock with practically no cohesion, which is flowing and has clayey minerals likely to produce swelling phenomena when they are exposed to atmospheric humidity or underground water.

1b. Mechanical defects of the rock

Fractures;
Fissures;
Bedding planes and schistosity;
Joints;
Fault planes and areas, fragmented or crushed areas;
Folds;
Voids.

Whatever the excavation method used, the fractures and fissures inherent in the rock mass are increased thereby reducing its strength characteristics.

2. State of stress

As a result of excavating the tunnel, the state of triaxial confinement of the unexcavated core, between D and 2D in front of the face, decreases, which means that ruptured areas in the core could develop. On the other hand, the excavation of a volume of rock produces a redistribution of stresses, increasing it in the nearby support and in the rock mass in front of the face, which leads to deformation and possible fracture.

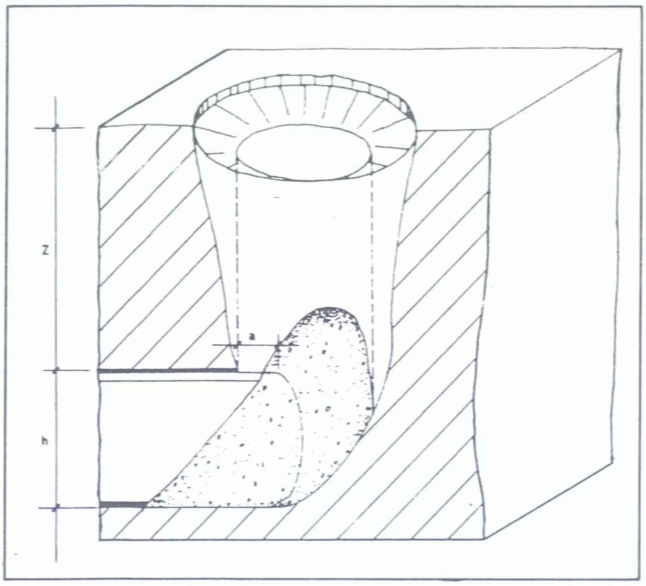


Fig 1. Chimney formation.

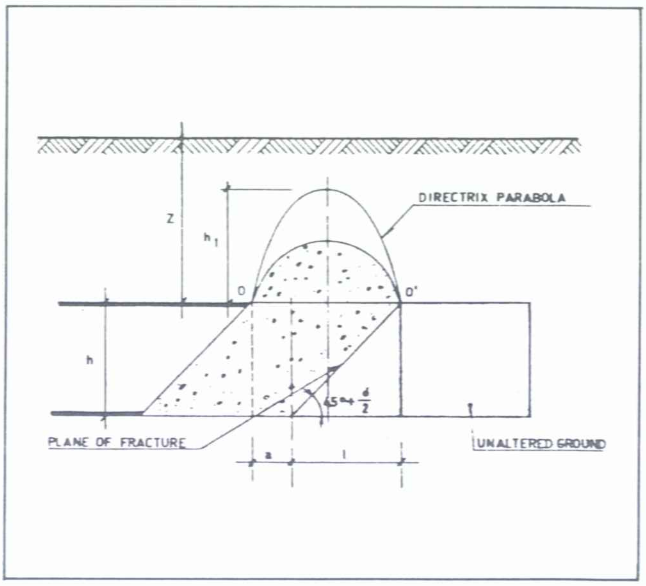


Fig 2. Defihition of the Paraboloid – Directrix Parabola.

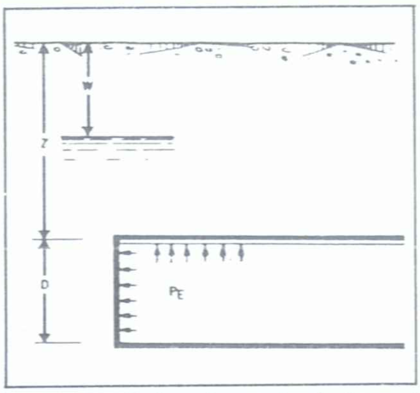


Fig 5. Tunnel below phreatic level subjected to a stabilisation pressure P_E.

stability index proposed by Broms and Bennermark:

$$N = \frac{\sigma z - \sigma t}{C_u} \quad (8)$$

for clayey soils (Fig 6)

The following cases will now be considered:

Grounds considered as isotropic and homogeneous and soft rock (T-1)	Soils and soft rock with cohesive-frictional behaviour (CF). Granular soils without cohesion (F). Clayey soils (C).
Stratified ground and soft rock with strength properties varying according to the depth. (T-2)	Soils and soft rock with cohesive-frictional behaviour (CF). Granular soils without cohesion (F). Clayey soils (C).

T-1 CF grounds

Homogeneous cohesive - frictional grounds belong to this group. From equations (6) and (9) of Table 1 the safety factor of the face is determined. Sometimes the stability of the prism (3) is more precarious than the set of three prisms, so the safety factor of the latter should be determined, considering the smallest

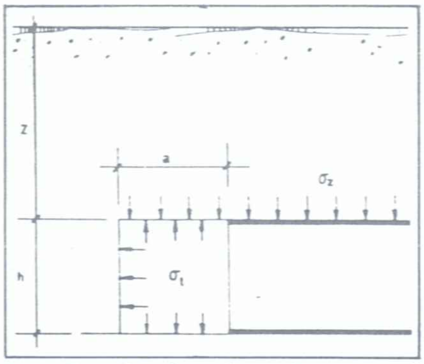


Fig 6. Tunnel in clay subjected to a vertical pressure σ_z and stabilisation σ_t.

Table 1a. T-1 CF ground

			Advance length without support		
Safety — factor of the face	General case — general safety factor A ≠ 0	$$FSF = \frac{\left[\frac{2(\tau_{m2}\ \tau_{m3})}{(1+a/l)^2} + 2\tau_{m3}\right]\frac{h_1}{b} + \frac{2\tau_{m3}}{(1+a/1)\sqrt{KA}}\frac{h_1}{h} + \frac{3,4\,C}{(1+a/1)^2\sqrt{KA}}}{\left[1 + \frac{2\,h}{3\,Z(1+a/1)^2}\right][\gamma Z - P_E]}$$ (6)	Maximum	a_m	$$\frac{b}{\frac{1,5\gamma Zb}{2\tau_{m3}\ h_1} - 1}$$ (10)
	Safety factor of the prism ③	$$FS_3 = \frac{2\,\tau_{m3}}{\gamma Z - P_E}\left[\cdot\frac{h_1}{b}\right]\left[1 + \frac{b}{a}\right]$$ (9)	Adopted	$a_m/2$	$$\frac{b}{\left[\frac{1,5\gamma Zb}{2\tau_{m3}\ h_1} - 1\right]2}$$ (11)

Parameters	Behaviour			
	Elastic	Elasto-plastic	Fracture	Brittle fracture
C	$C > 0.3\gamma Z$	$0.3\gamma Z > C > 0.15\gamma Z$	$C \leqslant 0.15\gamma Z$	$C < 0.3\gamma Z$
FSF	$FSF \geqslant 2$	FSF = 1.5 settlement — normally admissible. $1.3 \leqslant FSF < 1.5$ important settlement	$FSF \leqslant 1$	$FSF \geqslant \frac{3.14C}{\gamma Z}$ no long-term fracture FSF > 2 support not necessary FSF = 1.25 no short term — fracture
K_o	$K_o = \frac{\sigma h}{\sigma V}$; $K_o \begin{cases} K = 0 \text{ very shallow tunnels} \\ K = 0.5 \\ K = 1 \text{ very deep tunnels} \end{cases}$			
K_A	$1 \geqslant K_A \geqslant 0.5$			

Pressure which the support must withstand
$$P_S = \gamma Z\,\frac{\tau_{m3}\ h_1}{b}$$ (12)

Shape of the section	
Competent ground	Open
Not very competent ground	With invert floor
Loose ground	Circular

Tunnel	
Deep	Shallow
$Z/D \geqslant 3$ $h_1 = 1,7b$	$Z/D < 3$ $h_1 = z$

Table 1b. T-1 CF ground

Value of parameters	Deep Tunnels	Shallow tunnels
τ_{m3}	$\tau_{m3} = C + \{0.25[w\gamma + (Z - h_1 - w)(\gamma - \gamma_w)] - u\}tg\varphi$ (13)	$\tau_{m3} = C$ (14)
τ_{m2}	$\tau_{m2} = C + \frac{K_o}{2}\left[W_\gamma + (Z - h_1 - W)(\gamma - \gamma_w) + 3.4\,C/\sqrt{K_A} - \frac{(\gamma - \gamma_w)h}{2}\right]$ (15)	$\tau_{m2} = C + \frac{K_o}{2}\left[3.4C/\sqrt{K_A} - \frac{(\gamma - \gamma_w)h}{2}\right]$ (16)

T-2 C grounds

Stratified cohesive ground belongs to this group, whose geomechanical properties vary with depth. The expression of the safety factor (37) and the average cohesion expressions of the ground to be excavated, $\overline{C}1$, and of that above the crown, $\overline{C}_2$, are included in Table 4.

To determine (FSF) correctly, it will be necessary to determine the functions $f_{1(c)}$ and $f_{2(c)}$ which represent the variation of cohesion of ground with depth; in two groups — that of ground to be excavated, represented by an average cohesion value $\overline{C}_1$, and that of ground lying above the crown to a height h_1, represented by the average value of its cohesion $\overline{C}_2$.

Practical example

This example studies the stability at the face of a sub-aqueous tunnel which crosses a compact clay formation (Fig 8). The data are:

Diameter, D = 8m

Height of ground over the crown Z = 25m

Depth Z_W = 100m

Cohesion of ground, c = 0.015 Mpa = 15 T/m^2.

Unit weight γ = 1.6 T/m^3

Unit weight of sea water γ_Z = 1.025 T/m^3

The value of $\frac{Z}{D}$ = 3.125; the tunnel is therefore classified as a deep tunnel, since $\frac{Z}{D} \geqslant 3$.

The weight that gravitates over the tunnel, corresponding to the ground and to the water mass, will be given by the expression $\gamma Z = Z_W \gamma w + Z(\gamma - \gamma_w)$ (18); replacing the known values in this expression, we shall have: γZ = 100x1.025+25 (1.6-1.025) = 116.875 T/m^2.

Since the ground is cohesive, to determine the stability of the face, the expression will be used:

$$FSF = \frac{4\frac{h_1}{D} + 3.4}{1 + \frac{D}{3Z}} \left[\frac{C}{\gamma Z - P_E}\right] \quad (32)$$

We have h_1/D = 1.7m

Introducing the known parameters in (32) we shall have:

$$FSF = \frac{4x1.7+3.4}{1 + \frac{8}{3x25}} \left[\frac{15}{116.875 - P_E}\right]$$

On the basis of fixing a value of FSF = 1.3, we shall have the stabilisation pressure which the excavator shield would have to withstand, which should be:

P_E = 10.52 T./m^2.

Conclusion

Through the formulation proposed a study can be undertaken of stability of the excavation face within a broad range of situations and types of ground through which it is planned to construct a tunnel.

Starting from the characteristic parameters of the heterogeneous stratified ground [ζm^2, ζm^3, c, $\sqrt{KA}$, γ, γw, φ, u] and of the parameters of the situation and surroundings of the tunnel [Z, Z_w, W], and fixing a value for the safety factor (FSF), the following can be determined:

Table 2. T-2 CF ground

Value of parameters	Deep tunnels	Shallow tunnels
τ_{m3}	$\tau_{m3} = \overline{C}_2 + \{0.25[W\overline{\gamma}_3 + (Z - h_1 - W)(\overline{\gamma}_1 - \gamma_w)] - U\}tg\,\varphi_m$ (19)	$\tau_{m3} = \overline{C}_2$ (20)
τ_{m2}	$\tau_{m2} = \overline{C}_2 + \frac{K_o}{2}\left[W\overline{\gamma}_3 + (Z - h_1 - W)(\overline{\gamma}_1 - \gamma_w) + 3.4\,\overline{C}_1/\sqrt{K_A} - \frac{(\overline{\gamma}_2 - \gamma_w)h}{2}\right]$ (21)	$\tau_{m2} = \overline{C}_2 + \frac{K_o}{2}\left[3.4\,\overline{C}_1/\sqrt{K_A} - \frac{(\gamma_2 - \gamma_w)h}{2}\right]$ (22)
$\overline{\gamma}_1$ Average unit weight of ground between h_1	$\overline{\gamma}_1 = \frac{\sum \gamma_i Z_i}{\sum Z_i}$ (23)	
$\overline{\gamma}_2$ Average unit weight of ground to be excavated	$\overline{\gamma}_2 = \frac{\sum \gamma_i Z_i}{\sum Z_i}$ (24)	
$\overline{\gamma}_3$ Average unit weight of ground above the WTL	$\overline{\gamma}_3 = \frac{\sum_{i=m-n+1}^{i=n} \gamma_i Z_i}{\sum_{i=m-n+1}^{i=n} Z_i}$ (25) γ_i, Z_i Unit weight and thickness of each layer; n = number of layers; m = number of layers below the WTL	
$\overline{\gamma}_4$ Average unit weight of ground below the WTL	$\overline{\gamma}_4 = \frac{\sum_{i=1}^{i=m-n+1} \gamma_i Z_i}{\sum_{i=1}^{i=m-n+1} Z_i}$ (26)	
γZ	$\gamma Z = W\overline{\gamma}_3 + (Z - W)(\overline{\gamma}_4 - \gamma_w)$ (27)	
ϕ_m Average angle of friction	$\varphi_m = \frac{\sum \phi_i Z_i}{\sum Z_i}$ (28)	
$\overline{C}_2$ Average cohesion of ground above the crown	$\overline{C}_2 = \int_h^{h+h_1} f_2(c)\, d_y/h_1$ (29)	
$\overline{C}_1$ Average cohesion of ground to be excavated	$\overline{C}_1 = \int_o^h f_1(c)\, d_y/h$ (30)	

Table 3. T-1 C ground

Safety factor of the face	E Tamez	General case $a \neq 0$ General safety factor	$FSF = \dfrac{2\left[1 + \dfrac{1}{1+a/h} \cdot \dfrac{b}{h}\right] \dfrac{h_1}{b} + \dfrac{3.4}{(1+a/h)^2}}{\left[1 + \dfrac{2h}{3Z(1+a/h)^2}\right]} \left[\dfrac{C}{\gamma Z - P_E}\right]$ (31)
		$a = 0$ Circular tunnel $h = b = D$	$FSF = \dfrac{4\dfrac{h_1}{D} + 3.4}{1 + \dfrac{D}{3Z}} \left[\dfrac{C}{\gamma Z - P_E}\right]$ (32)
		Safety factor of the prism (3)	$FS_3 = \dfrac{2C}{\gamma Z - P_E} \left[\dfrac{h_1}{b}\left(1 + \dfrac{b}{a}\right)\right]$ (33)
	A Ellstein	$a \neq 0$ Circular tunnel $FSF = N_e \times \dfrac{C}{\gamma Z}$	$FSF = \dfrac{\left[2 + \dfrac{2+\sqrt{2}}{1+a/D}\right]}{\left[K_o\left(\dfrac{\gamma - \gamma_w}{\gamma} + \dfrac{\gamma_w W}{\gamma Z}\right) + \dfrac{D}{2Z} + \dfrac{\gamma - \gamma_w}{6\gamma Z}\dfrac{D}{Z} - \dfrac{a}{6Z} + \dfrac{\gamma_w}{\gamma}\left(1 - \dfrac{W}{Z}\right) - \dfrac{P_E}{\gamma Z}\right]} \left[\dfrac{C}{\gamma Z}\right]$ (34)
		$a = 0$	$FSF = \dfrac{[4 + \sqrt{2}]}{K_o\left(\dfrac{\gamma - \gamma_w}{\gamma} + \dfrac{\gamma_w W}{\gamma Z}\right) + \dfrac{D}{2Z} + \dfrac{\gamma - \gamma_w}{6\gamma Z}\dfrac{D}{Z} + \dfrac{\gamma_w}{\gamma}\left(1 - \dfrac{W}{Z}\right) - \dfrac{P_E}{\gamma Z}} \left[\dfrac{C}{\gamma Z}\right]$ (35)
	Broms Bennermark	Stability index	$N = \dfrac{\sigma_Z - \sigma_t}{C_u}$ (36) For circular tunnels in clay the short-term stability is obtained for $N \leqslant 5$

the stable excavation dimensions [b, h], the advance length without support [am] and the stabilisation pressure (P_E) which is necessary to apply against the face. The pressure which the support (PS) has to carry to guarantee stability of the tunnel can also be determined. □

Nomenclature used

Cohesion: c (t./m^2.)
Normal pressure regarding fracture plane: σ(t/m^2)
Interstitial or pore pressure: u (t/m^2)
Internal friction angle: φ (sexagesimal degrees).
Unit weight of water: γ_w (t/m^3)
Drop in hydraulic head: h(m)
Course corresponding to the drop in hydraulic head: 1(m)
Length advanced without support: a(m)
Protodyakonov factor: f
Tunnel height: h(m)
Tunnel width: b(m)
Uniaxial strength of ground σ_c (kg/cm^2)
Thickness of ground over crown: Z(m)
Height of ground which gravitates over the crown: h_1 (m)
Shearing stress in the prism (2): ζm^2 (t/m^2)
Shearing stress in the prism (3): ζm^3 (t/m^2)
Coefficient at rest: K_A
Pressure which gravitates over the tunnel crown: γZ (t./m^2.)
Stabilisation pressure: P_E (t/m^2)
Tunnel diameter: D (m)
Coefficient: $K_O = \frac{h}{v}$
Thickness of ground above the water table: W (m)
Vertical pressure over the crown: σ_z (t/m^2)
Horizontal pressure against the face: σ_t (t/m^2)
Vertical stress of field: σv
Horizontal stress of field: σh
Depth: A_w (m)

References

1. Tamez, E; Stability of tunnels excavated in soils, Mexico 1985.
2. Ellstein, A R. Heading failure of lined tunnels in soft soil, *Tunnels and Tunnelling*, June '86
3. Pera, J; Tunnelling in soft and water-bearing grounds, Lyon 1984
4. Juarez, E, Badillo, A. Rico Rodriguez Mexico 1976.

Table 4. T-2 C ground

Safety factor of the face	General case $a \neq 0$ General safety factor	$FSF \dfrac{2\,\bar{C}_2\left[1 + \dfrac{1}{1+a/h} \cdot \dfrac{b}{h}\right] \dfrac{h_1}{b} + \dfrac{3.4\,\bar{C}_1}{(1+a/h)^2}}{[\gamma Z - P_E]\left[1 + \dfrac{2h}{3Z(1+a/h)^2}\right]}$ (37)
	Parameters	
	$\bar{C}_1$ Average cohesion of ground to be excavated	$\bar{C}_1 = \int_o^h f_1(c)\, d_x/h$ (38)
	$\bar{C}_2$ Average cohesion of ground above the crown	$\bar{C}_2 = \int_h^{h_1+h} f_2(c)\, d_x/h_1$ (39)

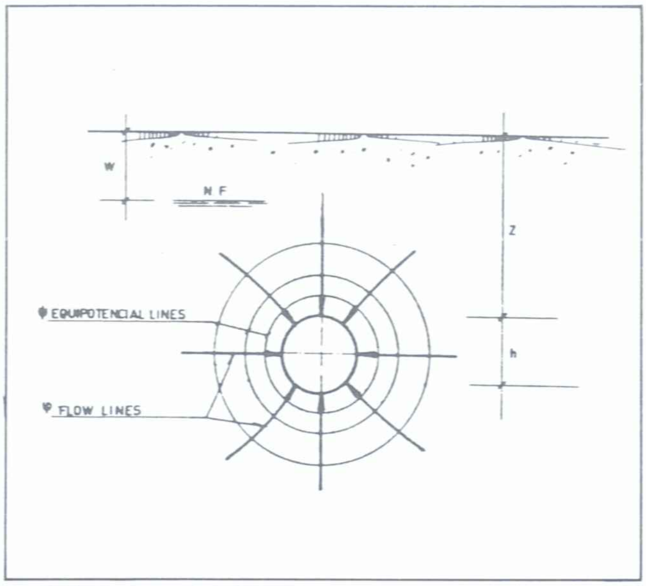


Fig 7. Equipotential and flow lines.

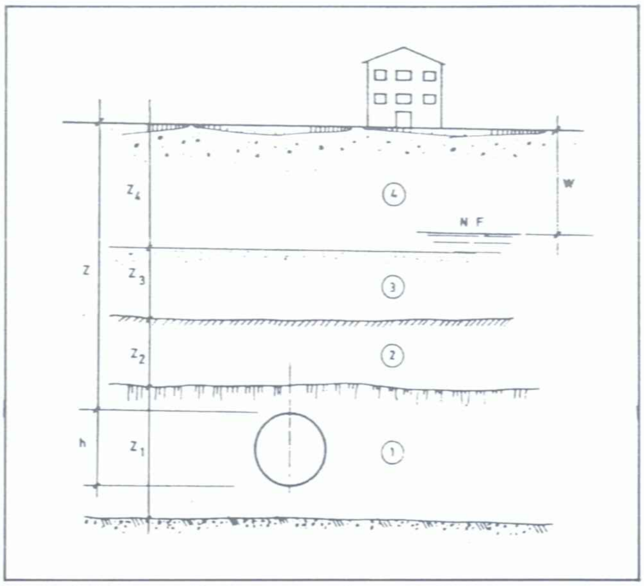


Fig 9. Urban tunnel below water table with different ground.

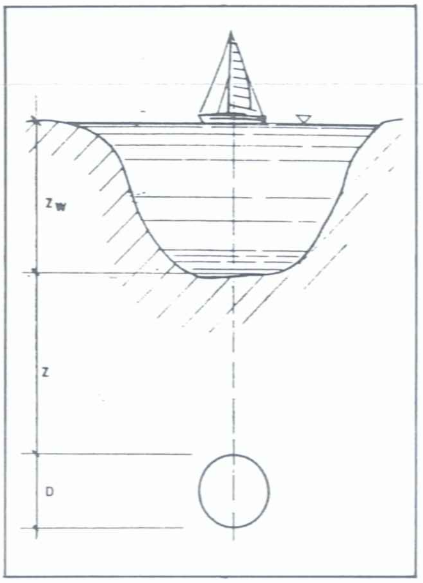


Fig 8. Subaqueous tunnel.

value for stability purposes.

The parameters ζm^3 and ζm^2 are the average values of the shearing strength of the ground acting on the faces of prisms (3) and (2) respectively.

The pore or interstitial pressure is important for determining the value of ζm^3, which is why this should be determined if an accurate stability study is to be made.

When tunnels are below the water table (Fig 7), it will be necessary to consider seepage forces because of the destabilising effect they introduce. Gravel, sand, silt or their combination become extremely unstable under the influence of seepage forces due to the rapid decrease in their shearing strength and the tendency to erosion. Seepage flow-rates are usually considerable and having to cope with them can cause difficulty during construction.

To avoid this, the following measures can be adopted:

a) Lowering the local water table
b) Applying pressure to the tunnel face with compressed air (PE) with a pressure greater than the hydrostatic pressure $(Ph) = [h + Z - W]\, \gamma w$.
c) Using closed front shields with pressure against the face.

The general expression for γZ in equations (6) and (9) is:

$$\gamma Z = W\gamma + (Z\text{-}W)\,(\gamma\text{-}\gamma w) \qquad (17);$$

w = Z for a tunnel above the water table. For sub-aqueous tunnels, the expression will be:

$$\gamma Z = Z_W \gamma w + Z\,(\gamma\text{-}\gamma_W) \qquad (18)\ (\text{Fig } 8).$$

In Table 1 the expressions are given for determining the parameters ζm^3, ζm^2, for deep and shallow tunnels, and values (c) and (FSF) are given for ground exhibiting elastic, elasto-plastic, fracture and brittle fracture properties. The K_O, K_A values are also indicated, as well as the advance lengths without support, and the pressures which the support should carry.

T-1 F ground

For granular soil without cohesion, such as: sand, silt, gravel or mixtures of them, the same expressions contained in Table 1 are applicable, taking into account the values of the following parameters:

Parameter	Value
C	0
K_A	0.5

T-2 CF ground

Cohesive-frictional ground belongs to this group, such as: silty sand, silty gravel, clayey sand, whose geomechanical properties vary with depth. In Fig 9 the excavation of a tunnel is shown in ground (1) with a thickness Z_1, with other ground (2), (3), (4) lying on top, with thicknesses (Z_2), (Z_3), (Z_4) with different geomechanical properties.

In Table 2 the expressions formulated generally are summarised which, introduced in expressions (6) or (9), will determine the safety factor of the face. Both the average cohesion in ground to be excavated and of that above the crown whose expressions are indicated, are taken into account, together with the average unit weights of the ground, to be excavated, and the ground both above and below the water table.

In expression (27) the generalised form of the weight of all the ground above the crown of the tunnel is indicated. To obtain these data, it is necessary to define the thickness of the different ground as well as its geomechanical parameters, together with the accurate measurement of the water table levels.

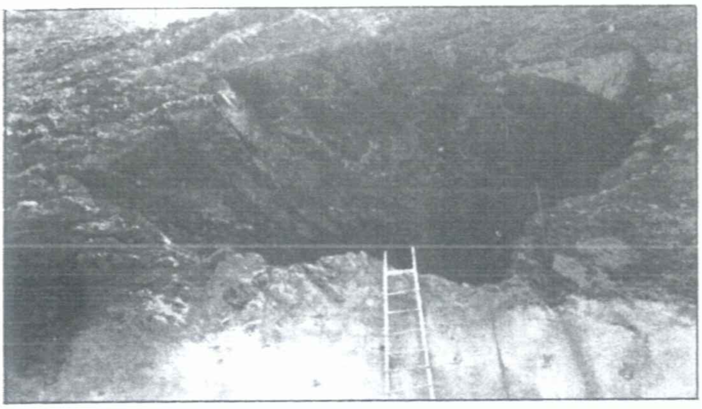
T-1 C ground

Homogeneous cohesive ground belongs to this group, such as: clay, silty clay and sandy silt. Expressions are included in Table 3 for determining the safety factor of the face according to different authors[1-2]. Expressions (31), (32) and (33) are particular cases of general expressions (6) and (9) for $l = h$, $K_A = 1$, $\zeta m^3 = \zeta m^2 = c$. For $a = 0$ the expression (32) is obtained.

Expression (34) is the generalised form of the expression proposed by A Ellstein[2] for a sub-aqueous tunnel (Fig 8), with diameter D, to which a stabilisation pressure P_E is applied.

Expressions (34) and (35) are valid for $W \leqslant Z - D/2$; for $W \geqslant Z$, $\gamma w = 0$; for intermediate values of W, namely: $Z \geqslant W > Z - D/2$, intermediate values of γ should be considered, between $(\gamma\text{-}\gamma w)$ and γ.

When calculating the safety factor of the face, all the expressions (31), (32), (33), (34), (35), (36) should be used and the results obtained with each of them compared.



Formation of the chimney which affects the surface.

As a result of these processes and the interstitial or pore pressure, the ground core can lose its initial strength, producing a flow of the material towards the void.

3. Presence of water

In cases of instability associated with flow and/or swelling of the ground, the interstitial water housed in the pores, fissures and fractures of the rock plays a vital role. The pressure of the water trapped in the pores and fissures (pore pressure) plays a very important role in the shearing fracture process which is quantified through the equation:

$$\tau = C + (\sigma - u) \operatorname{tg} \phi \qquad (1)$$

The relaxation of the rock core through loss of confinement causes small movements of interstitial water towards the excavation, giving the rock an additional amount of water which encourages the flow and/or swelling phenomena.

When, besides, there is free water which reaches a given level on the floor of the tunnel, the hydrostatic pressure and the hydrodynamic pressure should be considered, expressed as $P_{HD} = \gamma w \Delta h$ (2), which is capable of altering the specific weight of the material submerged and of reducing the effective pressure and hence the shearing strength when a flow of water occurs.

This flow, generated as a result of the variation in the head (pressure load + position load), induces seepage forces

$F_F = \gamma_{wi}$ (3), in which

$$i = \frac{\Delta h}{\Delta l} \quad (4)$$

is the hydraulic gradient.

These forces are tangential to the flow lines and represent an additional destabilising force.

Evolutional process of the fracture

Where instability occurs in the face, if controlling action is not taken, ground movements will progress affecting an increasingly large volume of rock. A state of equilibrium will eventually be reached, but at the expense of having formed a 'chimney' above the crown and a fracture in the core on the plane of maximum shearing (Fig 1).

The volume of rock gravitating to the crown of the tunnel will be that of the paraboloid defined by the directrix parabola in Fig 2 in which

$$00' = a + l \,;\; h_1 = \frac{B}{2f} \qquad (5)$$

and the parabola in Fig 3 in which HI = B.

The parameters that intervene are:
a = length advanced without support
f = Protodyakonov factor

for rocks $f = \frac{\sigma_c}{100}$

for soils: when $c = 0$, $f = \operatorname{tg}\phi$
When $c \neq 0$, $f = \frac{c}{\sigma_c} + \operatorname{tg}\varphi$

$l = h \operatorname{tg}(45° - \phi/2)$
$B = b + 2h \operatorname{tg}(45° - \phi/2)$
b = width of tunnel
h = height of tunnel
σ_c = simple compressive strength of the ground
c = cohesion
z = thickness of ground over the crown

This means that:

for $h_1 < Z$, $h_1 = \frac{B}{2f}$

for $h_1 > Z$, $h_1 = Z$

When the fracture reaches the surface, a subsidence crater will be produced, as observed in the photograph.

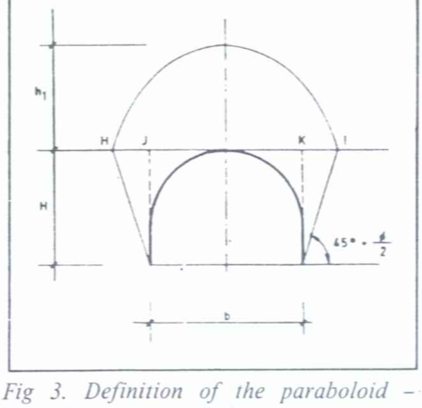


Fig 3. Definition of the paraboloid – Protodyakonov parabola.

Criteria for determining the instability of the face

There are several calculation models for determining the stability or otherwise of the face. They are all based, in one way or another, on the arching theory, proved experimentally, combined with the theory of elasticity. It is taken into account that above the crown, from a given height, the material does not gravitate over it, forming the known dome effect.

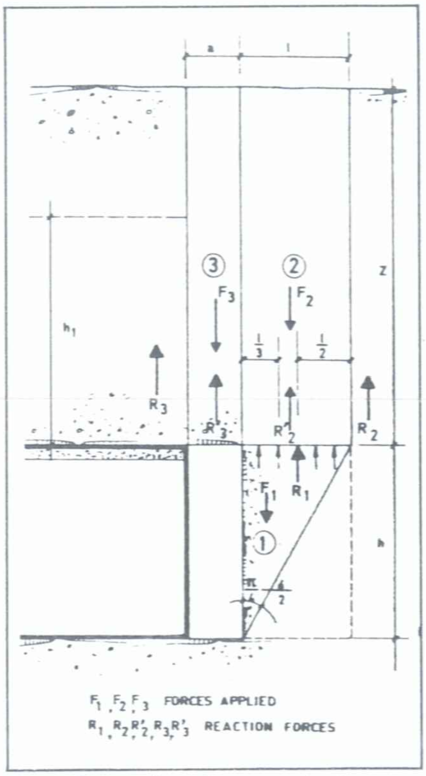


Fig 4. Prismatic volume gravitating over the crown system of forces.

To simplify the calculation, the paraboloid is replaced by a prismatic volume (Fig 4) in which the forces acting are established and a safety factory is defined of the stability of the face

$$FSF = \frac{M\Sigma FR}{M\Sigma FA},$$

through the ratio between the moments of the reaction forces and the forces applied, reaching the general expression proposed by E Tamez[1].

$$FSF = \frac{\left[\frac{2(\tau_{m2}\ \tau_{m3})}{(1 + a/l)^2} + 2\tau_{m3}\right]\frac{h_1}{b} + \frac{2\tau_{m3}}{(1 + a/l)\sqrt{KA}}\frac{h_1}{h} + \frac{3,4\,C}{(1 + a/l)^2\sqrt{KA}}}{\left[1 + \frac{2h}{3Z(1 + a/l)^2}\right][\gamma Z - P_E]} \qquad (6)$$

For tunnels in homogeneous and cohesive soils, in which lining accompanies excavation, the following general expression is also proposed, based on that defined by A Ellstein[2] (Fig 5).

$$FSF = \frac{\left[2 + \frac{2 + \sqrt{2}}{1 + a/D}\right]\left[\frac{c}{\gamma z}\right]}{\left[K_0\left(\frac{\gamma - \gamma w}{\gamma} + \frac{\gamma w W}{\gamma Z}\right)\frac{+D}{2Z} + \frac{\gamma - \gamma w}{6\gamma Z}\frac{D}{Z} - \frac{a}{6Z} + \frac{\gamma w}{\gamma}\left(\frac{1 - W}{Z}\right) - \frac{PE}{\gamma Z}\right]} \qquad (7)$$

Another equation frequently used is the